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# **Understanding Scaling Relations In Fracture and Mechanical Deformation of Single Crystal and Polycrystalline Silicon By Performing Atomistic Simulations at Mesoscale**

**Han Sung Kim & Vikas Tomar**

**Aerospace and Mechanical Engineering**

**University of Notre Dame, Notre Dame-Indiana, USA, 46556**

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- Limitations of MD simulations

- **Method and Framework**

- Equivalent crystal lattice method

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# Limitations of MD simulation

- **Discrepancy between MD simulation and experimental results**

1. **Length Scale**

- Recent billion atoms simulation ( $1 \times 1 \times 1 \mu\text{m}$  for metal)
- Most of MD simulation is nanometer scale (In many case, using only a small part of actual specimen for simulations)

2. **Time scale**

- Most MD simulation's time step is femto second
- Current time step extending methods can increase time step about factor of 15

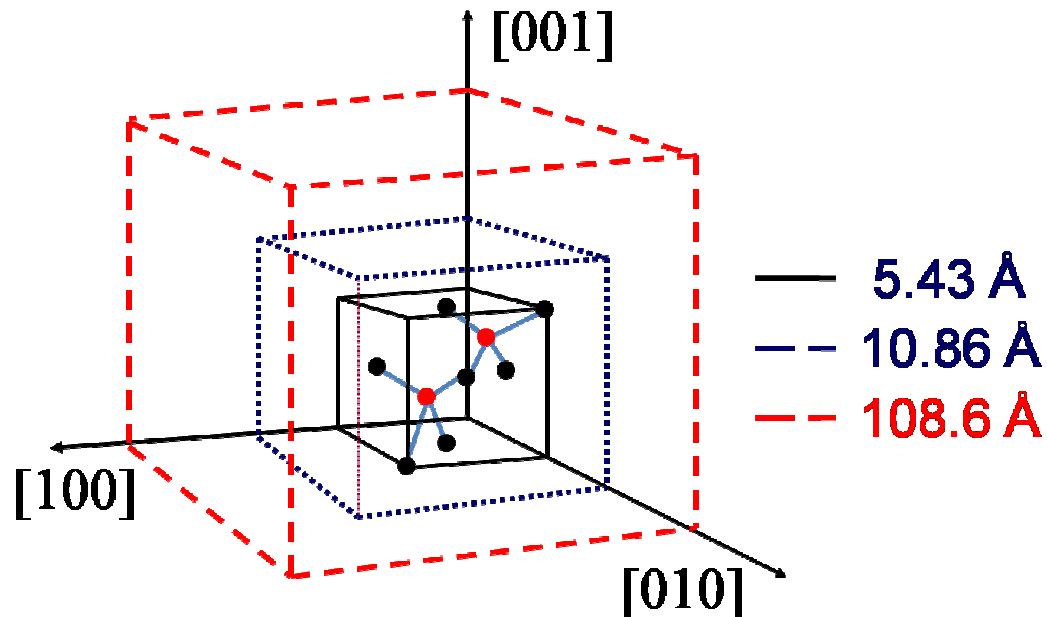
- **Necessity to increase length and time scale of MD simulations**

# Method and framework

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- **Equivalent crystal lattice method**
- **New inter-atomic potential parameters**
  - Tersoff potential (T3)
- **Comparison of properties**
  - C11, C12, C44, Bulk modulus, Total lattice energy.
- **Time step calculations**
  - Maxwell-Boltzman distribution
- **Simulation details**
  - Construction of suprtcells, NST ensemble,

# Equivalent crystal lattice method



- Lattice parameters:  
5.43, 10.86, 108.6, and  
1086 Å

-Number of atoms in the  
unit cell are the same

--Fraction coordinates in  
the unit cell are the same

-Potential parameters are  
found by minimizing  $F$

$$F = \sum_{i=1}^{N_{obs}} w_i (f_i^{tar} - f_i^{calc})^2$$

$N_{obs}$  is the number of observables,  $f^{tar}$  and  $f^{calc}$  are the target and calculated values of the observables, and  $w_i$  is the weight factor for the given observables

# Parameters for Tersoff potentials

Lattice Constant	5.43Å (T3)	10.86Å	108.6Å	1086Å
A (eV)	1830.8	9911.8	9682272.8	12256245358.6
B (eV)	471.18	2500	2500000.0	2846954605.4
$\lambda_1$ (Å <sup>-1</sup> )	2.4799	1.2125	0.1195	0.01235
$\lambda_2$ (Å <sup>-1</sup> )	1.7322	0.8166	0.08071	0.008282
$\alpha$	0	0	0	0
$n$	0.78734	0.78734	0.78734	0.78734
$\beta$	1.0999E-06	1.0999E-06	1.0999E-06	1.0999E-06
$n$	0.78734	0.78734	0.78734	0.78734
$c$	100390	100390	100390	100390
$d$	16.218	16.218	16.218	16.218
$h$	-0.59826	-0.59826	-0.59826	-0.59826

$$E = \sum_i E_i = \frac{1}{2} \sum_{i \neq j} V_{ij} ,$$

$$V_{ij} = f_C(r_{ij}) [a_{ij} f_R(r_{ij}) + b_{ij} f_A(r_{ij})]$$

$$f_R(r) = A \exp(-\lambda_1 r) ,$$

$$f_A(r) = -B \exp(-\lambda_2 r) ,$$

$$f_C(r) = \begin{cases} 1, & r < R - D \\ \frac{1}{2} - \frac{1}{2} \sin \left[ \frac{\pi}{2} (r - R) / D \right], & R - D < r < R + D \\ 0, & r > R + D \end{cases} \quad (1c)$$

$$b_{ij} = (1 + \beta^n \xi_{ij}^n)^{-1/2n} ,$$

$$\xi_{ij} = \sum_{k (\neq i, j)} f_C(r_{ik}) g(\theta_{ijk}) \exp[\lambda_3^3 (r_{ij} - r_{ik})^3] , \quad (1d)$$

$$g(\theta) = 1 + c^2/d^2 - c^2/[d^2 + (h - \cos\theta)^2] ,$$

$$a_{ij} = (1 + \alpha^n \eta_{ij}^n)^{-1/2n} ,$$

$$\eta_{ij} = \sum_{k (\neq i, j)} f_C(r_{ik}) \exp[\lambda_3^3 (r_{ij} - r_{ik})^3] . \quad (1e)$$

## Results comparison

Properties	Target values (experimental)	Tersoff (T3)	New potential			
		Lattice parameters(Å)				
		5.43 Å	10.86 Å	108.6 Å	1086 Å	
C <sub>11</sub> (GPa)	165.7	142.5 (14)	137.0 (17)	134.86 (18.6)	136.6 (17.5)	
C <sub>12</sub> (GPa)	63.9	75.4 (18)	70.0 (9.7)	68.13 (6.63)	73.5 (15)	
C <sub>44</sub> (GPa)	79.6	69 (13.3)	67.42 (15)	67 (15.8)	64.8 (18.6)	
B (GPa)	98	98 (0.0)	98.5 (0.6)	90.37(7.7)	94.5 (3.5)	
Lattice E (ev)	-37.36 (5.43Å) -298.88 (10.86Å) -298880 (108.6Å) -298880000 (1086Å)	-37.04(0.9)	-289.4 (0.14)	298387 (0.1)	-298935530(0.05)	
Atomic mass	28.0855	28.0855	224.684	224684	224684000	

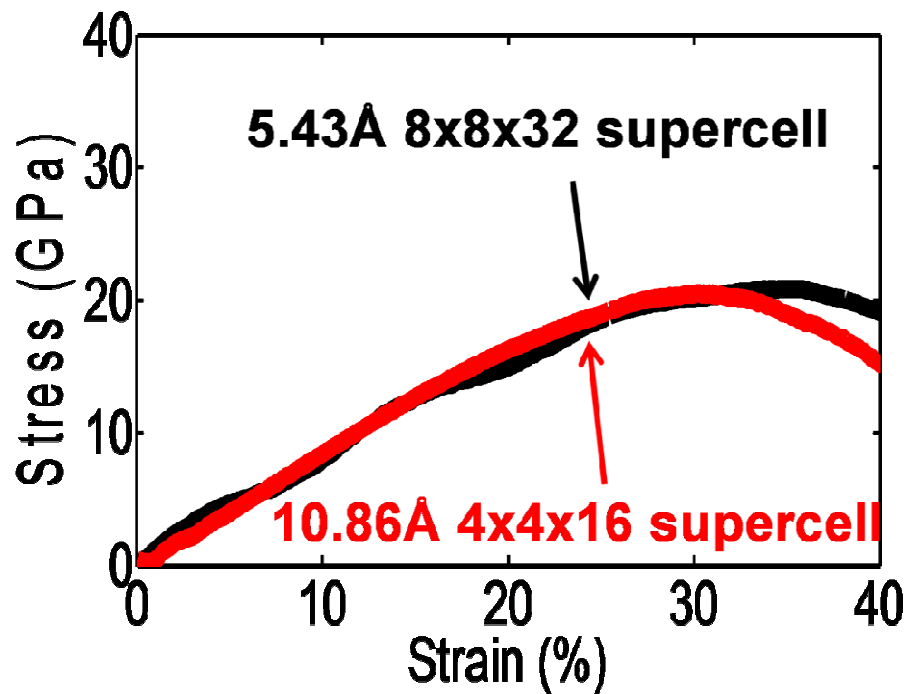


## Silicon nanowires Simulation detail

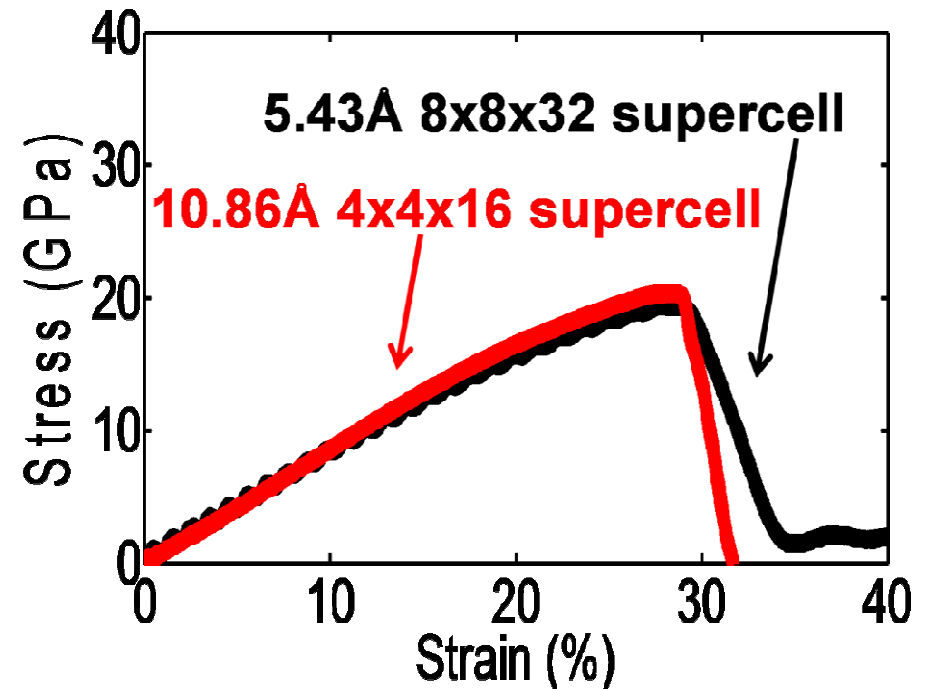
Supercell size	5.43 Å	10.86 Å	108.6 Å	1086 Å
4x4x32	2.17x2.17x17.36 nm (4096)	4.34x4.34x34.8 nm (4096)	43x43x348 nm (4096)	0.43x0.43x3.48μm (4096)
8x8x32	4.34x4.34x17.36 nm (16384)	8.7x8.7x34.8 nm (16384)	87x87x348 nm (16384)	0.87x0.87x3.48μm (16384)
16x16x32	8.7x8.7x17.36 nm (65536)	17.4x17.4x34.8 nm (65536)	174x174x348 nm (65536)	1.74x1.74x3.48μm (65536)

- Applying tensile load constant strain rates:  
**0.01 %/step and 0.001 %/step**
- NST ensemble (Constant particles, temperature, stress)
- Temperature : 300K, Nose-Hoover thermostat

# Simulation results between 5.43 and 10.86Å equivalent lattice



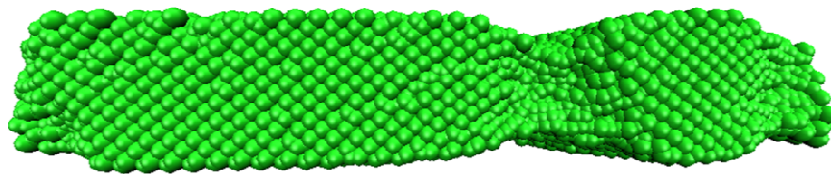
(a) strain rate 0.01%/step



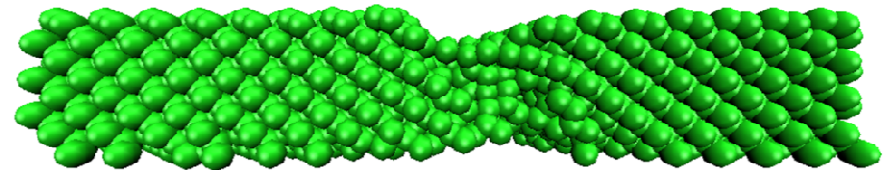
(b) strain rate 0.001%/step

Dimension: 4.34 x 4.34 x 17.36 nm

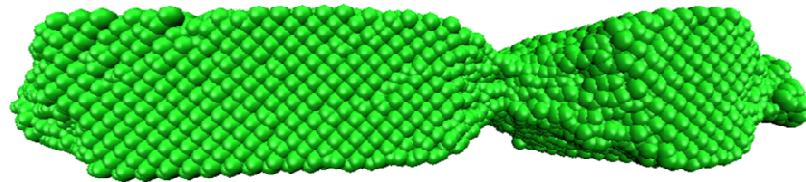
# Fracture results between 5.43 and 10.86Å equivalent lattice



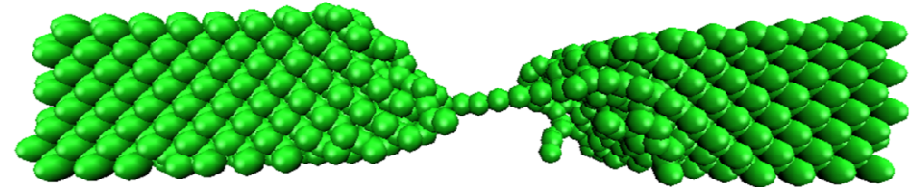
**8x8x32 at 32%**



**4x4x16 at 30%**



**8x8x32 at 34%**



**4x4x16 at 32%**

(a)

(b)

**5.43 Å**

**10.86 Å**

**Dimension: 4.34 x 4.34 x 17.36 nm**

**strain rate 0.001 %/step**

# Statistical analysis: Coefficient of correlation (R)

$$R = \sqrt{\frac{SSR}{SSTO}} = \sqrt{1 - \frac{SSE}{SSTO}}$$

$SSR = \sum (\hat{Y}_i - \bar{Y})^2$  : Regression sum of square,  
where  $\bar{Y}$ : mean value,  $\hat{Y}_i$ : value from the fitted line

$SSE = \sum (Y_i - \hat{Y}_i)^2$  : Error sum of square

$SSTO = SSE + SSR$  : Total sum of square

Properties	Predictor variables			
	Diameter	Surface area	Volume	Surface to volume ratio
Young's modulus	0.14	0.10	0.15	-0.74
Fracture stress	0.30	0.25	0.26	-0.85

**- The surface to volume ratio is most critical factor**

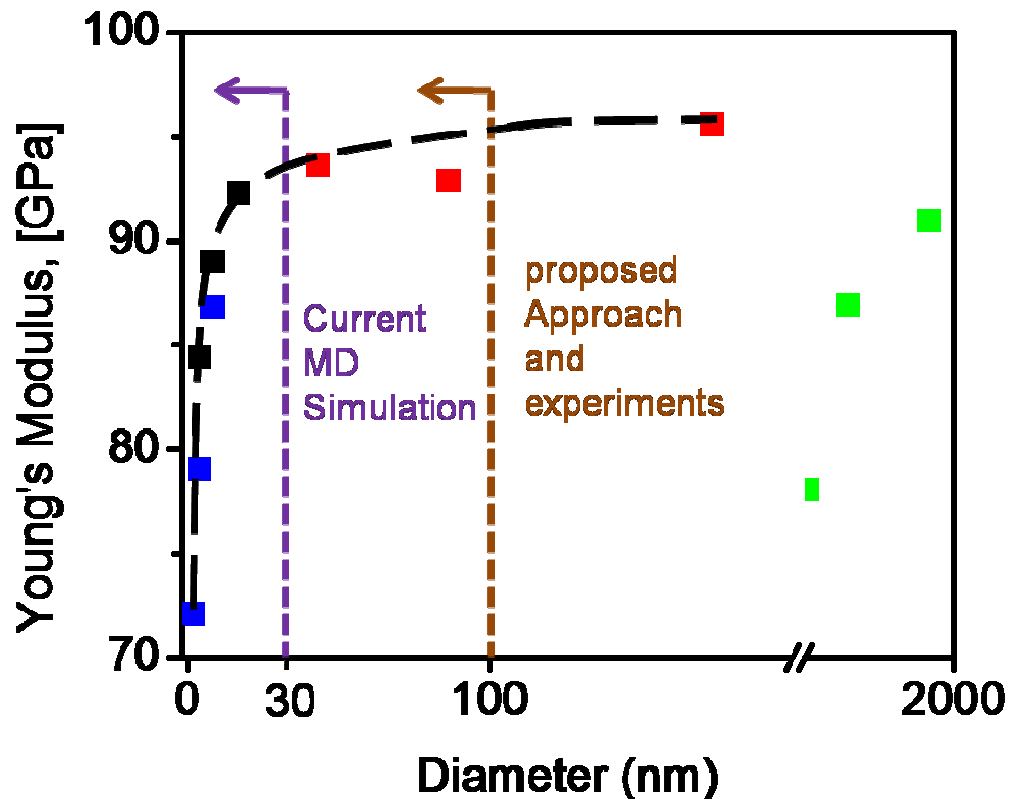
# Young's modulus transition diameter (Si nanowire)

## Discrepancy b/w current MD and experiments

-Current MD predicts 4-30nm for transition diameter (Park et al., MRS bulletin, 34, 2009)

-Experiments measured 100nm for transition diameter (Li et al. Appl.Phys.Lett, 83,2003 and Han et al. adv. Mater.,19, 2007)

-Our simulation predicted around 100nm for transition diameter

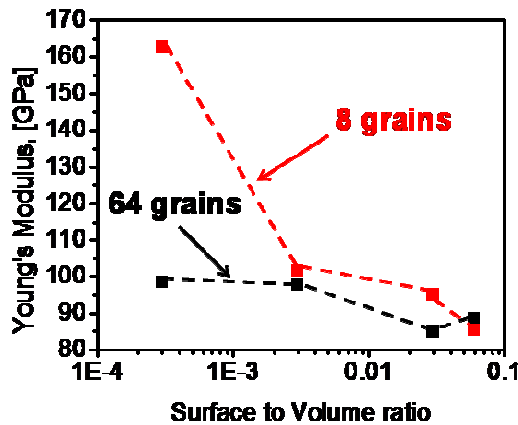


## Polysilicon simulation detail (Number of atoms and sizes)

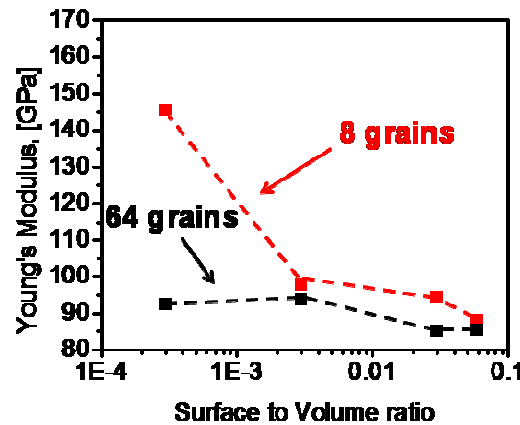
	5.43 Å	10.86 Å	108.6 Å	1086 Å
8 grains	49356 (101x101x101 Å)	49356 (202x202x202 Å)	49356 (2020x2020x2020 Å)	49356 (20200x20200x20200 Å)
64 grains	48058 (101x101x101 Å)	48058 (202x202x202 Å)	48058 (2020x2020x2020 Å)	48058 (20200x20200x20200 Å)

- **Constructed bulk polysilicon and surfaced polysilicon**
- **Applying tensile load with constant strain rates:  
0.01%/step and 0.001%/step**
- **NST ensemble (Constant particles, temperature, stress)**
- **Temperature : 300K, Nose-Hoover thermostat**

# Number of grain and surface effect on properties



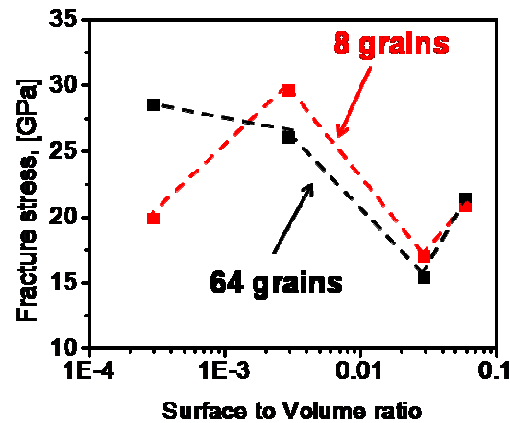
(a)



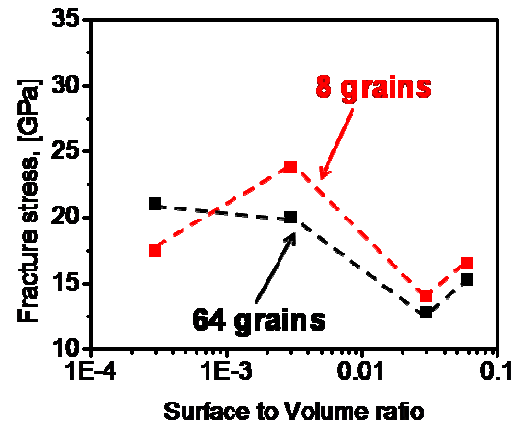
(b)

(a) Young's modulus with strain rate 0.01 %/step

(b) Young's modulus with strain rate 0.001 %/step



(c)



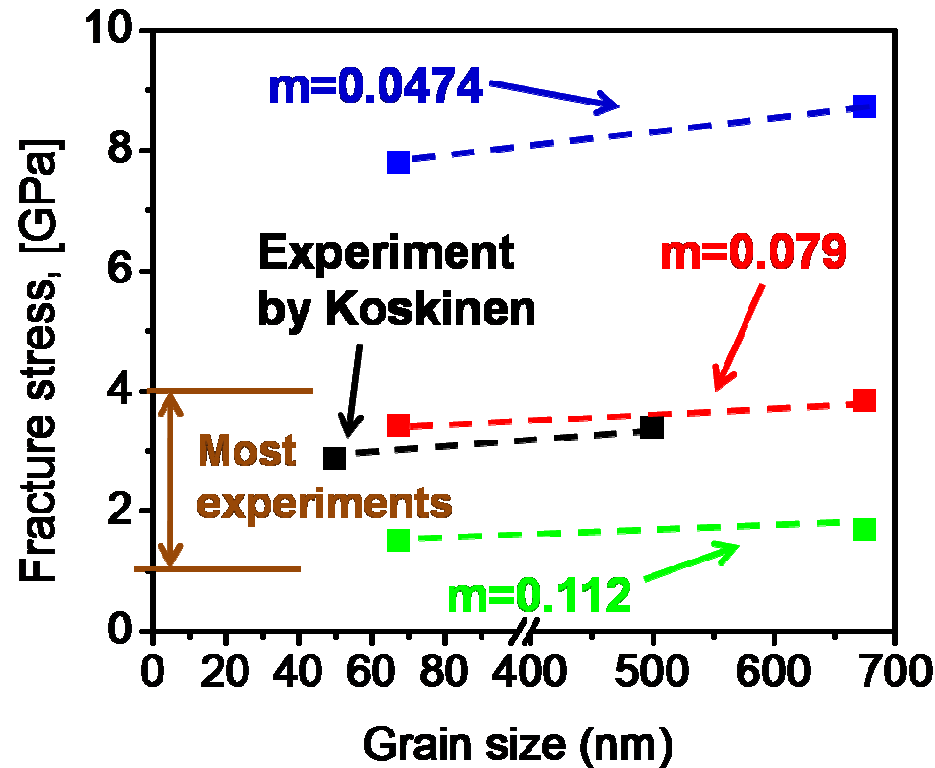
(d)

(c) fracture stress with strain rate 0.01 %/step

(d) fracture stress with strain rate 0.001 %/step

- With the same surface to volume ratio (same dimension), number of grain changes the Young's modulus: Grain size effect

# Comparison with experiments



-Koskinen et al. measured fracture stress with various grain size

- Inverse Hall-Petch mechanism

- Our results agree with Koskinen's experiments

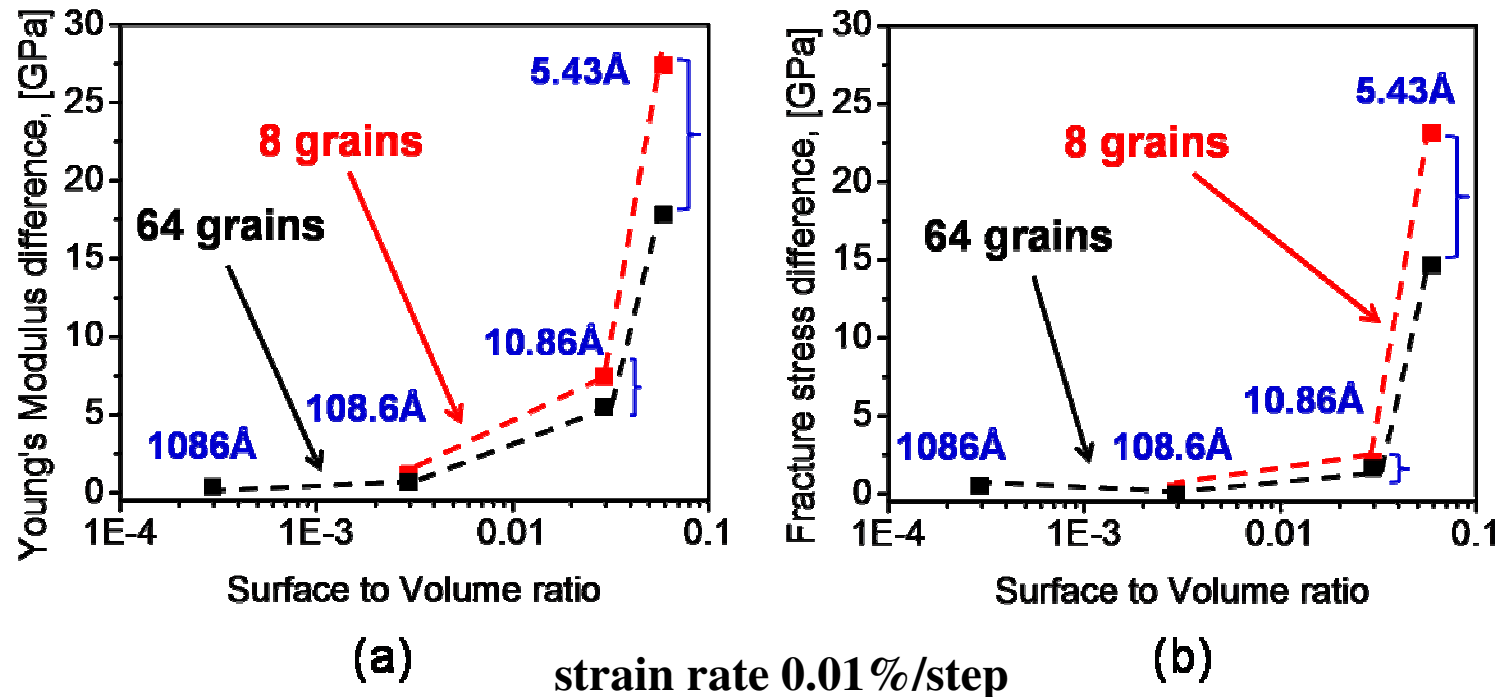
- Our fracture stresses are converted assuming typical experimental strain rate 2E-3/sec

$$\frac{\sigma}{\sigma_0} = \left( \frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right)^m$$

$\sigma$  : stress,  $\dot{\epsilon}$  : strain rate,  $m$  : strain rate sensitivity index  
(  $0.0474 \leq m \leq 0.112$ , average  $m = 0.079$  )

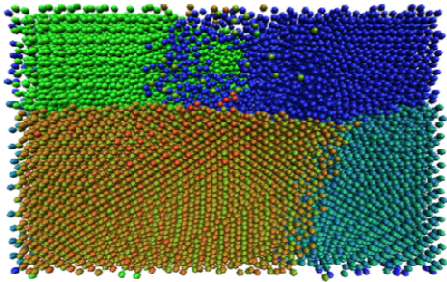


# Limit of surface and grain size effects on polysilicon

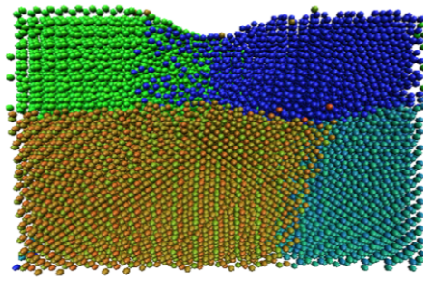


- Grain size effect and surface effect is decreasing and converge to 0 !!
- About  $1 \mu\text{m}$  of the grain size and  $2^3 \mu\text{m}^3$  of specimen size are the limit above which the Young's modulus and fracture stress is not influenced by grain size as well as surface for polysilicon
  - Agree with experiments by Fancher et al. , J. Mater. Sci., 36, 2001,

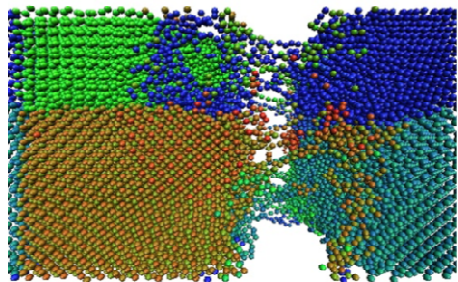
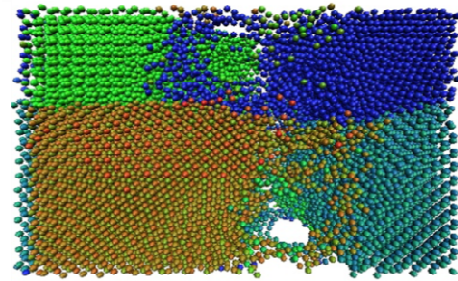
# Fracture transition from brittle to ductile with surfaces



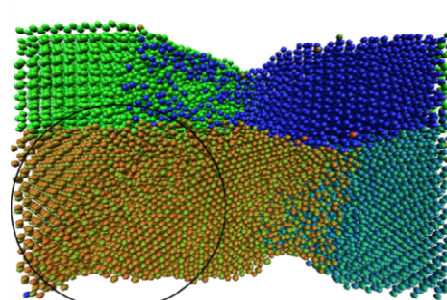
(a) bulk polysilicon



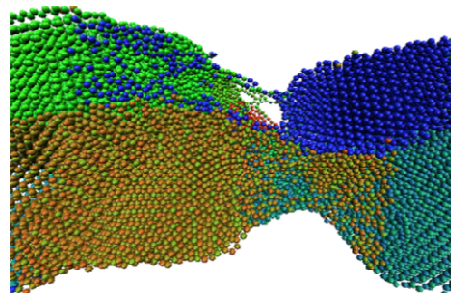
(b) surfaced polysilicon



(a)



Slip induced



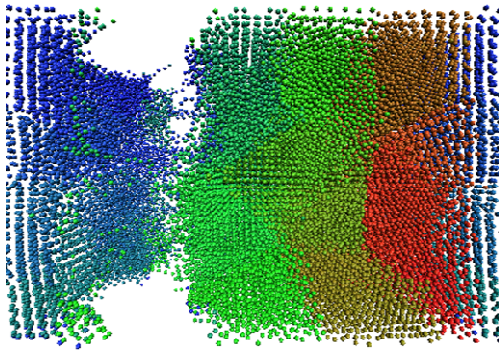
(b)

101 x 101 x 101 Å 8 grain polysilicon  
with strain rate 0.001 %/step

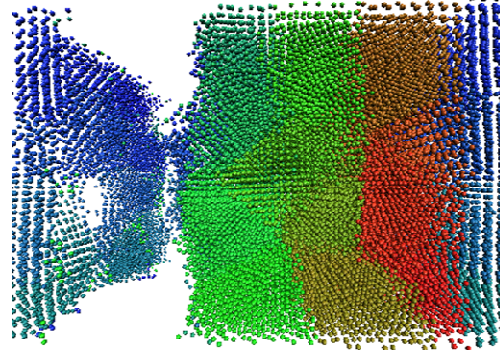
-Surfaces make materials  
more ductile when compared  
to bulk materials

-Slip induced fracture is  
observed inside grain

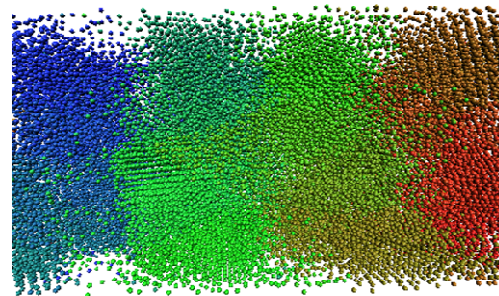
# Fracture behaviors for equivalent lattices



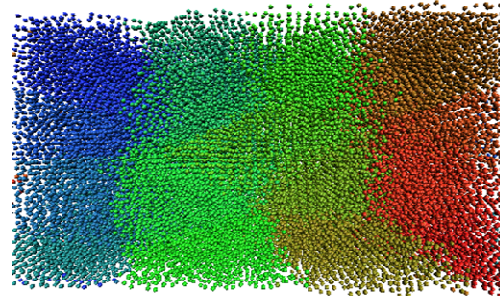
(a)



(b)



(c)



(d)

- (a) 5.43 Å lattice
- (b) 10.86 Å equivalent lattice
- (c) 108.6 Å equivalent lattice
- (d) 1086 Å equivalent lattice

64 grain bulk polysilicon with  
strain rate 0.001 %/step

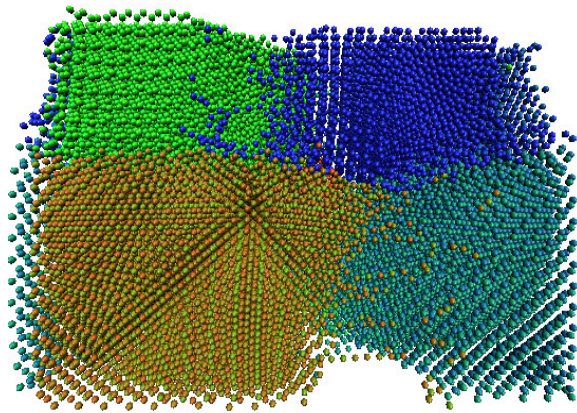
-Fracture initiation,  
propagation, final pattern are  
identical b/w 5.43 Å and 10.86 Å

-Fracture initiations are  
identical for all lattices

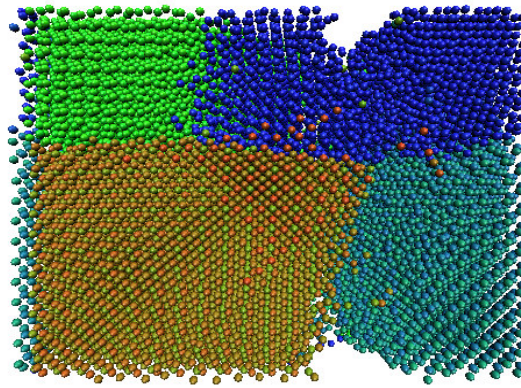
-Temperature s are higher than  
300K for 108.6Å and 1086Å



# Intergranula vs. transgranula depending on strain rate



(a)



(b)

Sz2 gs1sr1 and sr2 -  
surface

202 x 202 x 202 Å 8 grain  
surfaced polysilicon with  
strain rate with 10.86Å  
equivalent lattice  
(a) 0.01%/step (b)  
0.001%/step

-Fracture propagation  
depends on strain rate

-Mostly, intergranula

-Transgranula  
observed

# Conclusion

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- **By using equivalent crystal lattice method,**
  - Time step increased about factor of 500  
(From 0.001 to 0.5fs)
  - Length scale increased factor of 200  
(5.43 Å to 1086 Å)
- **The surface to volume ratio is the most important factor at nanometer and micron scale**
- **MD simulation results are more close to experiments if actual specimen dimension is used for MD simulations**
  - Young's modulus transition diameter (Si nanowire)

# Conclusion

- MD simulations of polysilicon with grain sizes ranging from 3.4nm to 1.3  $\mu\text{m}$  are made possible using equivalent lattices
- Inverse Hall-Petch mechanism is observed for polysilicon
  - Agrees with experiments, opposite to metals and alloys
- The Young's modulus is not influenced by grain size when grain size is larger than 7nm for bulk polysilicon
  - Verified by DFT calculations
- The Young modulus is strongly influenced by grain size for surfaced polysilicon
- The existence of surfaces make materials more ductile when compared to bulk materials